Ultraviolet Extension of a Model with Dynamical Electroweak Symmetry Breaking by Both Top-Quark and Technifermion Condensates

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We construct and analyze an ultraviolet extension of a model in which electroweak symmetry breaking is due to both technifermion and top-quark condensates. The model includes dynamical mechanisms for all of the various gauge symmetry breakings. We discuss certain aspects in which it requires additional ingredients to be more realistic.

I. INTRODUCTION

The origin of electroweak symmetry breaking (EWSB) continues to be an outstanding mystery. In one class of models this breaking is produced dynamically by means of an asymptotically free, vectorial, gauge interaction based on an exact gauge symmetry, commonly called technicolor (TC), that becomes strongly coupled on the TeV scale, causing the formation of bilinear technifermion condensates [1]. In order to communicate the electroweak symmetry breaking in the technicolor sector to the Standard-Model (SM) fermions (which are technisinglets), one embeds the technicolor symmetry in a larger theory called extended technicolor (ETC) [2]. A different approach is based on the idea that because of its large mass, the top quark should play a special role in electroweak symmetry breaking, and models of this type feature a top-quark condensate, $\langle \bar{t}t \rangle$. An early realization of this idea made use of (nonrenormalizable) four-fermion operators [3], while later renormalizable models used separate asymptotically free, vectorial SU(3) gauge interactions acting on the third generation of quarks and on the first two generations of quarks, denoted as SU(3)$_1$ and SU(3)$_2$, respectively [4, 5]. These are often called "topcolor" models. In these models the SU(3)$_1$ interaction becomes sufficiently strong, at a scale Λ_t of order 1 TeV, to produce the $\langle \bar{t}t \rangle$ condensate. The SU(3)$_1$ interaction actually treats the t and b quarks in the same way and hence, by itself, would also produce a $\langle \bar{b}b \rangle$ condensate equal, up to small corrections, to $\langle \bar{t}t \rangle$, and a resultant dynamical b-quark mass essentially equal to m_t. To prevent the formation of such a $\langle \bar{b}b \rangle$ condensate, these models include an additional set of hypercharge-type U(1)$_1$⊗U(1)$_2$ gauge interactions. In these models the SU(3)$_1$ ⊗ SU(3)$_2$ and U(1)$_1$⊗U(1)$_2$ symmetries each break to their respective diagonal subgroups, which are the usual color SU(3)$_c$ and weak hypercharge U(1)$_Y$ groups. There has been considerable interest in hybrid models that combine the properties of technicolor and topcolor and thus feature both technifermion condensates and a top-quark condensate [4]-[7]. These are often called "topcolor-assisted technicolor" or TC2 models. In these theories, most of the observed top-quark mass, $m_t \simeq 173$ GeV, is due to

the $\langle \bar{t}t \rangle$ condensate. Reviews of these models include [8]-[10].

In this paper we shall carry out an exploratory construction and analysis of an ultraviolet extension of a TC2 model in which we explicitly specify an embedding of the TC symmetry in a higher-lying ETC group and dynamical mechanisms for the necessary breakings of both the ETC group to the TC group and of the SU(3)$_1$⊗SU(3)$_2$⊗U(1)$_1$⊗U(1)$_2$ group to SU(3)$_c$⊗U(1)$_Y$. Because our model does not purport to be complete, we call it an ultraviolet extension rather than an ultraviolet completion. It is recognized that models that involve a top quark condensate and associated strong interactions of the top quark at a scale not too much larger than m_t are tightly constrained by the excellent agreement between the measured cross section for $p\bar{p} \to t\bar{t}X$ at $\sqrt{s} = 1.96$ TeV from the CDF and D0 experiments at the Fermilab Tevatron [11–13] and perturbative QCD predictions [14]. TC2 models are also subject to bounds from searches for colorons, top pions, $t\bar{t}$ resonances, and by constraints from precision electroweak data [4]-[9]. Detailed analyses of the phenomenology of TC2 models have been given in the literature [5]-[7]. Our present work is somewhat complementary to these analyses, in that we focus on the effort to build an ultraviolet extension of a TC2 model, although we do comment on some phenomenological implications of this extension. Before proceeding, it is appropriate to recall that TC2 models represent only one among many ideas for physics beyond the Standard Model; other ideas include, for example, a top-quark seesaw, supersymmetry, and theories involving higher spacetime dimensions, in particular, "higgless" models and string theory. Here we will concentrate on a (four-dimensional) TC2 approach.

This paper is organized as follows. In Section II we review some necessary background on TC/ETC and TC2 models. In Section III we discuss our ultraviolet extension and analyze its properties. Section IV contains a brief discussion of the consequences that would ensue if one tried to build a model including an SU(2)$_{L,1}$ ⊗ SU(2)$_{L,2}$ sector analogous to the SU(3)$_1$ ⊗ SU(3)$_2$ and U(1)$_1$ ⊗ U(1)$_2$ sectors of TC2 theories. In a concluding section, we summarize the successes of the model and cer-

tain problems that deserve further study. Some notation and formulas are contained in an Appendix.

II. SOME BACKGROUND

A. TC/ETC Models

Here we briefly review some relevant background on models with dynamical electroweak symmetry breaking, first on TC/ETC models and then on models featuring top-quark condensates. Early works on ETC tended to model ETC effects via four-fermion operators connecting SM fermions and technifermions, with some assumed values for their coefficients. More complete studies have taken on the task of deriving these four-fermion operators by analyses of renormalizable, reasonably ultraviolet-complete, ETC models. These models normally gauge the generational index and combine it with the technicolor index. Thus, given that the TC and ETC gauge groups are $SU(N_{TC})_{TC} \subset SU(N_{ETC})_{ETC}$, one has the relation

$$N_{ETC} = N_{gen} + N_{TC} , \qquad (2.1)$$

where $N_{gen} = 3$ denotes the number of oberved SM fermion generations [15]. The ETC gauge symmetry breaks in a series of stages, in one-to-one correspondence with the SM fermion generations, down to the residual exact technicolor symmetry. Some recent reviews include Refs. [9], [16]-[18]. At the highest breaking scale, denoted Λ_1, the first-generation fermions split off, and since they communicate with the EWSB technifermion sector only via ETC gauge bosons with masses of order Λ_1, it follows that their masses are the smallest. The reasonably ultraviolet-complete ETC models of Refs. [19]-[22] used the minimal non-Abelian value $N_{TC} = 2$ in order to reduce technicolor corrections to W and Z propagators. Accordingly, these models employed an $SU(5)_{ETC}$ group. In these models, the ETC-breaking scales corresponding to the three generations exhibit a hierarchy encompassing Λ_1 of order 10^3 TeV, an intermediate scale, Λ_2, and the smallest scale, Λ_3 of order a few TeV.

TC/ETC models in which the SM-nonsinglet fermions transform vectorially under the ETC gauge group are able to satisfy constraints from flavor-changing neutral-current processes. This was shown in Refs. [21, 22] to result from approximate residual generational symmetries. These models rely upon a slowly running (walking) TC gauge coupling associated with an approximate infrared zero of the TC beta function in order to enhance fermion masses [23, 24]. They also must rely upon this walking behavior in another way, namely that in the presence of a slowly running coupling at the TeV scale, small perturbations by SM gauge interactions have a magnified effect. Thus, although the $SU(3)_c$ coupling is small at this scale, it provides enhancement for the condensation of techniquarks, relative to technileptons, and hence causes the techniquark condensate to occur at a higher

scale, with the result that the dynamically induced techniquark masses are larger than the technilepton masses. This, in turn, can explain why the masses of the quarks are greater than the mass of the charged lepton in each generation. (The very small masses of neutrinos require a more complicated mechanism, involving a low-scale see-saw [20].) Furthermore, since the weak hypercharge interaction favors the condensation of the techniquarks of charge 2/3 in a one-family model, while inhibiting the condensation of techniquarks of charge $-1/3$ (see Eqs. (2.4)-(2.5)), the former naturally condense at a higher scale then the latter. This can explain why the charge 2/3 quarks of the higher two generations are heavier than the charge $-1/3$ quarks (explaining why $m_u < m_d$ presumably necessitates incorporating effects of off-diagonal elements of the respective up-quark and down-quark mass matrices). However, it is not clear that this effect is large enough to account for the large mass ratio m_t/m_b without violating custodial-symmetry constraints and, moreover, is able to produce realistic CKM mixing [22, 25]. Indeed, the large value of the top-quark mass was one of the main motivations for models featuring a $\langle \bar{t}t \rangle$ condensate.

B. Models with $\langle \bar{F}F \rangle$ and $\langle \bar{t}t \rangle$

We proceed to discuss some details of TC2 models that will be needed for the explanation of our ultraviolet extension. These use a gauge group,

$$G_{TC} \otimes G_{ASM} , \qquad (2.2)$$

where G_{TC} is the technicolor group and G_{ASM} is the augmented SM (ASM) group

$$G_{ASM} = SU(3)_1 \otimes SU(3)_2 \otimes SU(2)_L \otimes U(1)_1 \otimes U(1)_2 . \qquad (2.3)$$

Our notation for the running gauge couplings (with the scale μ implicit here) is g_{TC} and, for the five factor groups in G_{ASM}, g_{c1}, g_{c2}, g, g_1', and g_2'. The running squared couplings are denoted $\alpha_j \equiv g_j^2/(4\pi)$ for the various factor groups G_j. The gauge symmetry (2.2) is operative above a scale of order 1 TeV and below the lowest ETC breaking scale. As discussed above, the $SU(3)_1$ interaction couples to the third generation of quarks, while the $SU(3)_2$ interaction couples to the first two generations of quarks. The $SU(3)_1$ coupling at this scale is considerably stronger than the $SU(3)_2$ coupling, and, indeed, becomes strong enough to produce the $\langle \bar{t}t \rangle$ condensate.

To prevent the formation of a $\langle \bar{b}b \rangle$ condensate by this $SU(3)_1$ interaction, TC2 models rely on the $U(1)_1 \otimes U(1)_2$ factor group displayed in Eq. (2.3). In early TC2 models, the $U(1)_1$ and $U(1)_2$ interactions coupled, respectively, to SM fermions of the third generation, and to SM fermions of the first two generations, according to their weak hypercharges. Motivated by constraints from

precision electroweak data, more recent TC2 models [6, 7] have adopted a different set of $U(1)_1 \otimes U(1)_2$ charge assignments in which the $U(1)_1$ interaction couples in the same manner to all three generations, which are singlets under $U(1)_2$. These models have thus been characterized as having flavor-universal hypercharge. At the scale Λ_t, the $U(1)_1$ interaction is assumed to be strong enough to (i) enhance the formation of the $\langle \bar{t}t \rangle = \langle \bar{t}_L t_R \rangle + h.c.$ condensate, since the relevant hypercharge product

$$(-Y_{Q_L})Y_{u_R} = -\frac{4}{9} \qquad (2.4)$$

is attractive, and (ii) prevent the formation of a $\langle \bar{b}b \rangle = \langle \bar{b}_L b_R \rangle + h.c.$ condensate, since the hypercharge product

$$(-Y_{Q_L})Y_{d_R} = +\frac{2}{9} \qquad (2.5)$$

is repulsive [26]. However, there are several constraints on the strength of the $U(1)_1$ coupling. First, if it were too large, then there would be excessive violation of custodial symmetry. Second, since the $U(1)_1$ (as well as $U(1)_2$) gauge interaction is not asymptotically free, a moderately strong $U(1)_1$ coupling would bring with it the danger of a Landau pole at an energy not too far above the 1 TeV scale, so that the model could not be regarded as a self-consistent low-energy effective field theory. These constraints have been used in TC2 model-building [4]-[9]. Indeed, in view of these constraints and the fact that, as the energy scale μ decreases, the $U(1)_1$ coupling gets weaker while the $SU(3)_1$ coupling gets stronger, there is a rather limited set of values of couplings and a limited interval in which this scenario can take place in a self-consistent manner. In particular, the $SU(3)_1$ coupling at the scale Λ_t must be fine-tuned to be only slightly greater than the critical value for the formation of the $\langle \bar{t}t \rangle$ condensate, so that a rather weak $U(1)_1$ coupling can still prevent the formation of a $\langle \bar{b}b \rangle$ condensate [5]-[7].

To the extent that this top-quark mass generation by $SU(3)_1$ is analogous to the dynamical generation of constituent-quark masses in quantum chromodynamics (QCD), then, since the latter are of order Λ_{QCD}, one would infer that Λ_t would be roughly comparable to m_t. More quantitatively, one can use the approximate relation [27]

$$f_t^2 \simeq \frac{3 \Sigma_t^2}{4\pi^2} \ln \left(\frac{\Lambda_{int}}{\Sigma_t} \right) \qquad (2.6)$$

where Λ_{int} represents a cutoff scale characterizing the asymptotic decay of the dynamical mass Σ_t, considered as a running quantity. This scale, Λ_{int}, enters in the integral that one calculates in deriving this relation. Setting $\Sigma_t \simeq m_t$ and using the rough estimate $\Lambda_{int} \simeq 2\Sigma_t$, one obtains $f_t \simeq 70$ GeV. The technifermion condensation yields an analogous f_{TC}. Both the top-quark and technifermion condensates transform as $\Delta T_3 = 1/2$ under $SU(2)_L$ and $|\Delta Y| = 1$ under $U(1)_Y$, and hence produce

a W mass given, to leading order, by

$$m_W^2 = \frac{g^2(N_{TD}f_{TC}^2 + f_t^2)}{4} \qquad (2.7)$$

where N_{TD} denotes the number of $SU(2)_L$ technidoublets in the theory. Our ultraviolet extension will use a one-family TC model, so that $N_{TD} = (N_c + 1) = 4$. In the absence of the f_t^2 contribution from the top-quark condensate (i.e., in regular technicolor), Eq. (2.7) would yield $f_{TC} \simeq 125$ GeV; the value of f_{TC} in the TC2 theory is slightly reduced by the presence of the f_t^2 term. Since $f_t^2/(N_{TD}f_{TC}^2) \lesssim 0.1$, most of the contribution to the W mass in the TC2 model is provided by technicolor. Similar comments apply to the Z mass.

One should remark on a difference between the generation of dynamical masses for light quarks in QCD and techniquarks in TC, on the one hand, and the generation of the top-quark mass in TC2 theories, on the other hand. In the former two cases, the gauge interactions responsible for the condensates and resultant dynamical fermion masses are exact. In contrast, $SU(3)_1$ is broken at a scale comparable to the scale Λ_t where it gets strong and produces the $\langle \bar{t}t \rangle$ condensate. It is thus plausible that, to compensate for this, Λ_t should be somewhat larger than $\Sigma_t \simeq m_t$, say of order 1 TeV, and we will assume this approximate value here. Slightly below the Λ_t scale, the $SU(3)_1 \otimes SU(3)_2$ symmetry group breaks to its diagonal subgroup that treats all generations symmetrically, namely usual color $SU(3)_c$, while the $U(1)_1 \otimes U(1)_2$ symmetry group breaks to a diagonal subgroup, which is the usual weak hypercharge, $U(1)_Y$. Thus, one has the symmetry breaking $G_{ASM} \to G_{SM}$, where $G_{SM} = SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$.

We shall use the more modern type of TC2 model with a $U(1)_1$ coupling universally to all generations [6, 7] to serve as the basis for our ultraviolet extension. We display SM fermion representations in this type of model below. In our notation, the three numbers in parentheses are the dimensions of the representations of the three non-Abelian factor groups in G_{ASM}; the subscripts are the $U(1)_1$ and $U(1)_2$ hypercharges; and a and $a' \in \{1, 2, 3\}$ are $SU(3)_1$ and $SU(3)_2$ indices:

$$\begin{pmatrix} u^{a'} \\ d^{a'} \end{pmatrix}_L, \ \begin{pmatrix} c^{a'} \\ s^{a'} \end{pmatrix}_L \ : \quad 2(1,3,2)_{1/3,0} \qquad (2.8)$$

$$\begin{pmatrix} t^{a} \\ b^{a} \end{pmatrix}_L \ : \quad (3,1,2)_{1/3,0} \qquad (2.9)$$

$$u_R^{a'}, \ c_R^{a'} \ : 2(1,3,1)_{4/3,0}; \quad t_R^{a} : (3,1,1)_{4/3,0} \qquad (2.10)$$

$$d_R^{a'}, \ s_R^{a'} \ : 2(1,3,1)_{-2/3,0}; \quad b_R^{a} : (3,1,1)_{-2/3,0} \qquad (2.11)$$

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L, \ \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L \ : \quad 2(1,1,2)_{-1,0} \qquad (2.12)$$

$$\begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L \quad : \quad (1,1,2)_{-1,0} \qquad (2.13)$$

and

$$e_R, \ \mu_R \ : 2(1,1,1)_{-2,0}; \quad \tau_R \ : (1,1,1)_{-2,0} \ . \qquad (2.14)$$

It is an option whether one explicitly includes right-handed electroweak-singlet neutrinos, since they are singlets under G_{ASM}.

III. CONSTRUCTION AND ASSESSMENT OF AN ULTRAVIOLET EXTENSION

A. General Structure

We find that it is not possible to have the usual ETC structure given by Eq. (2.1). The reason is quite fundamental; the full ETC symmetry is incompatible with the essential feature of the model, namely the fact that the first two generations of SM fermions transform according to different representations of G_{ASM} than the third generation. In other words, the ETC symmetry implies, *a fortiori*, that unitary transformations that mix up the three left-handed $SU(2)_L$ quark doublets leave the theory invariant, but this requirement is incompatible with the assignment of the first two generations of these quark doublets to the representation $(1,3)$ of $SU(3)_1 \otimes SU(3)_2$ and the third to the different representation of this group, $(3,1)$. Similarly, the ETC symmetry implies, *a fortiori*, that unitary transformations that mix up the three generations of right-handed up-type quarks and, separately, the three generations of down-type quarks leave the theory invariant, but this requirement is incompatible with the assignment of the first two generations of these quark fields to the representation $(1,3)$ of $SU(3)_1 \otimes SU(3)_2$ and the third generation to the $(3,1)$ representation.

In view of this fundamental incompatibility, we shall construct the ETC group by embedding the first two generations of SM fermions together with the technifermions in ETC multiplets. Hence, for our ETC model, the relation (2.1) is altered to read

$$N_{ETC} = N_{gen} - 1 + N_{TC} = 2 + N_{TC} \ . \qquad (3.1)$$

As before, in order to minimize TC corrections to W and Z propagators, we again choose the minimal non-Abelian value, $N_{TC} = 2$, so our ETC group is $SU(4)_{ETC}$. We thus consider a model that, at a high scale, is invariant under the gauge symmetry

$$G = G_{ETCA} \otimes G_{ASM} \ , \qquad (3.2)$$

where

$$G_{ETCA} = SU(4)_{ETC} \otimes SU(2)_{HC} \otimes SU(3)_{MC} \otimes SU(2)_{UC} \qquad (3.3)$$

and G_{ASM} was given in Eq. (2.3). The group G_{ETCA} contains the ETC group, $SU(4)_{ETC}$, together with three

additional gauge interactions (the subscript A in $ETCA$ refers to these additional interactions): (i) hypercolor (HC) $SU(2)_{HC}$, which helps in the breaking of $SU(4)_{ETC}$ in two sequential stages, to $SU(3)_{ETC}$ and then to the residual exact technicolor group, $SU(2)_{TC}$; (ii) metacolor (MC) $SU(3)_{MC}$, which breaks $SU(3)_1 \otimes SU(3)_2$ to the diagonal subgroup, color $SU(3)_c$; and (iii) ultracolor (UC) $SU(2)_{UC}$, which breaks $U(1)_1 \otimes U(1)_2$ to the diagonal subgroup, weak hypercharge $U(1)_Y$. With the fermion content to be delineated below, all of the four gauge interactions in G_{ETCA} are asymptotically free.

The fermions with SM quantum numbers, including the usual SM fermions and the technifermions, are assigned to the representations displayed below. We use notation such that the four numbers in the parentheses are the dimensions of the representations of the group

$$SU(4)_{ETC} \otimes SU(3)_1 \otimes SU(3)_2 \otimes SU(2)_L \ , \qquad (3.4)$$

while the two subscripts are the hypercharges for the gauge groups $U(1)_1$ and $U(1)_2$, respectively. Since all of these fermions are singlets under the additional gauge interactions in G_{ASM}, namely $SU(2)_{HC} \otimes SU(3)_{MC} \otimes SU(2)_{UC}$, we do not include these factor groups in the listings. The index i is an $SU(4)_{ETC}$ index, with $i = 1,2$ referring to the first two generations and $i = 3,4$ being $SU(2)_{TC}$ gauge indices. As before, e.g. in [21], we use a compact notation in which $u^{a',1} \equiv u^{a'}$, $u^{a',2} \equiv c^{a'}$, $d^{a',1} \equiv d^{a'}$, $d^{a',2} \equiv s^{a'}$, $e^1 \equiv e$, and $e^2 \equiv \mu$. The fermion representations are

$$Q_L^{a',i} = \begin{pmatrix} u^{a',1} & u^{a',2} & u^{a',3} & u^{a',4} \\ d^{a',1} & d^{a',2} & d^{a',3} & d^{a',4} \end{pmatrix}_L \quad : \quad (4,1,3,2)_{1/3,0} \tag{3.5}$$

$$\begin{pmatrix} t^a \\ b^a_L \end{pmatrix}_L \quad : \quad (1,3,1,2)_{1/3,0} \qquad (3.6)$$

$$(u^{a',1}, u^{a',2}, u^{a',3}, u^{a',4})_R \quad : \quad (4,1,3,1)_{4/3,0} \qquad (3.7)$$

$$t^a_R \ : (1,3,1,1)_{4/3,0} \qquad (3.8)$$

$$(d^{a',1}, d^{a',2}, d^{a',3}, d^{a',4})_R \quad : \quad (4,1,3,1)_{-2/3,0} \qquad (3.9)$$

$$b^a_R \ : \quad (1,3,1,1)_{-2/3,0} \qquad (3.10)$$

$$L_L = \begin{pmatrix} \nu^1 & \nu^2 & \nu^3 & \nu^4 \\ e^1 & e^2 & e^3 & e^4 \end{pmatrix}_L \quad : \quad (4,1,1,2)_{-1,0} \qquad (3.11)$$

$$\begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L \quad : \quad (1,1,1,2)_{-1,0} \qquad (3.12)$$

$$(e^1, e^2, e^3, e^4)_R \ ; \quad (4,1,1,1)_{-2,0} \qquad (3.13)$$

and

$$\tau_R : \quad (1,1,1,1)_{-2,0} . \qquad (3.14)$$

As was alluded to above, this is thus a one-family technicolor model [28]. Given the motivation for the structure of the model, it is clear why there are no SM fermions that are simultaneously nonsinglets under both $SU(3)_1$ and $SU(3)_2$. As pointed out above, one cannot embed all of the generations of each type of fermion in a single corresponding ETC multiplet, since there is an incompatibility between the essential ETC feature of treating the three generations in a symmetric manner at the high scale and the fact that in these types of models the first two generations are subject to different gauge symmetries than the third generation. Hence, with the present embedding in $SU(4)_{ETC}$, there is only mixing of the first two generations with each other, but no full three-generation CKM (Cabibbo-Kobayashi-Maskawa) mixing. Thus, $V_{ub} = V_{cb} = V_{td} = V_{ts} = 0$. (Indeed, the observed CKM quark mixing represents the difference in mixings between the up-quark and down-quark sectors, so three-generational mixings in these individual sectors are necessary but not sufficient to fit the observed CKM mixing.) The fact that the third-generation quarks transform differently under $SU(3)_1 \otimes SU(3)_2$ than the first two generations of quarks was recognized in early TC2 model-building to pose a challenge to getting full CKM mixing [4, 5], and this problem manifests itself directly in our UV extension. This shows that further ingredients are required for a satisfactory larger ultraviolet completion.

B. Generalities on Fermion Condensation Channels

In general, in an asymptotically free gauge theory involving possible condensation of fermions transforming according to the representations R_1 and R_2 of the gauge group G_j to a condensate transforming as R_{cond}, an approximate measure of attractiveness of this channel

$$R_1 \times R_2 \to R_{cond} \qquad (3.15)$$

is

$$\Delta C_2 = C_2(R_1) + C_2(R_2) - C_2(R_{cond}) , \qquad (3.16)$$

where $C_2(R)$ is the quadratic Casimir invariant for the representation R [29]. If several possible condensation channels are possible, it is expected that condensation occurs in the most attractive channel (MAC), i.e., the one with the largest value of ΔC_2. For a vectorial gauge interaction, the most attractive channel is $R \times \bar{R} \to 1$, producing a condensate in the singlet representation of the gauge group (with $\Delta C_2 = 2C_2(R)$) and thus preserving the gauge invariance. In this case, as the reference energy scale μ decreases from large values where the gauge interaction is weak, this condensation is expected to occur when $\alpha_j(\mu)C_2(R)$ exceeds a value of order unity. Some

results relevant to this are given in the Appendix. For a particular asymptotically free gauge interaction, one must check to see whether, given its (light or massless) fermion content, it will evolve from high scales where it is weakly coupled to lower scales in a manner that leads to a growth in the coupling that is sufficient to trigger fermion condensation, or whether, alternatively, its coupling could approach an infrared fixed point that is too small for such condensation to occur. In the latter case, this gauge interaction would not spontaneously break its chiral symmetries. In the model studied here, it is required that the $SU(2)_{HC}$, $SU(3)_1$, $SU(3)_{MC}$, $SU(2)_{UC}$, and $SU(2)_{TC}$ gauge interactions produce various condensates, and we will show that this is, indeed, consistent with the sets of nonsinglet fermions subject to these respective interactions. Throughout our analysis, it is understood that there are theoretical uncertainties inherent in analyzing such strong-coupling phenomena as fermion condensation.

We proceed to describe the fermion contents of the rest of the model. Since the full model is a chiral gauge theory, it follows that the Lagrangian describing the physics at a high scale $\sim 10^4$ TeV has no fermion mass terms. An analysis of global symmetries is of interest especially since some of these symmetries are broken by condensates produced by gauge symmetries that become strong at various lower energy scales. We shall discuss these global symmetries below.

C. $SU(2)_{HC}$ Sector

We shall need a set of fermions whose role is to break the $SU(4)_{ETC}$ symmetry in two stages down to $SU(2)_{TC}$. These fermions are singlets under all of the gauge symmetries except $SU(4)_{ETC} \otimes SU(2)_{HC}$, so we only list their dimensionalities under these two groups:

$$\psi_{j,R} : \quad (\bar{4}, 1) \qquad (3.17)$$

$$\chi_R^{j,\alpha} : \quad (4, 2) \qquad (3.18)$$

and

$$\zeta_R^{jk,\alpha} : \quad (6, 2) , \qquad (3.19)$$

where j and α are $SU(4)_{ETC}$ and $SU(2)_{HC}$ indices, respectively. The 6-dimensional representation of $SU(4)$ is the antisymmetric rank-2 tensor representation, $\square$, which is self-conjugate (and hence has zero $SU(4)_{ETC}$ gauge anomaly).

We next discuss the global flavor symmetries involving hypercolor-nonsinglet fermions. The fact that the $\chi_R^{j,\alpha}$ and $\zeta_R^{jk,\alpha}$ fields are nonsinglets under two interactions, namely $SU(4)_{ETC}$ and $SU(2)_{HC}$, that become strongly coupled at comparable scales ($\Lambda_1 \sim \Lambda_{HC} \sim 10^3$ TeV) plays an important role in the determination of this global chiral symmetry. In the hypothetical limit

where, at a given scale, the $SU(2)_{HC}$ coupling were imagined to be much stronger than the $SU(4)_{ETC}$ coupling, it would follow that the sector of HC-nonsinglet fermions would be invariant under the classical global flavor symmetry group $U(4)_\chi \otimes U(6)_\zeta$, or equivalently, $SU(4)_\chi \otimes SU(6)_\zeta \otimes U(1)_\chi \otimes U(1)_\zeta$. Here, the global $U(1)_\chi$ and $U(1)_\zeta$ transformations are defined to rephase $\chi_R^{j,\alpha}$ and $\zeta_R^{jk,\alpha}$, respectively. Both of these global U(1) symmetries are broken by $SU(2)_{HC}$ instantons, with one linear combination remaining unbroken. Just as $SU(2)_L$ instantons break quark number, N_q, and lepton number, $N_L \equiv L$, but preserve $N_c N_q - N_L = B - L$, so also the $SU(2)_{HC}$ instantons preserve the linear combination $4N_\chi - 6N_\zeta$, or equivalently, $2N_\chi - 3N_\zeta$. Let us denote the corresponding global (g) number symmetry as $U(1)_{HCg}$. Thus, if $SU(4)_{ETC}$ interactions could be neglected relative to $SU(2)_{HC}$, then the actual global chiral symmetry group of this sector would be $SU(4)_\chi \otimes SU(6)_\zeta \otimes U(1)_{HCg}$. However, although the $SU(2)_{HC}$ interaction is stronger than the $SU(4)_{ETC}$ interaction, the latter is never negligible, and hence the global flavor symmetry group is not $SU(4)_\chi \otimes SU(6)_\zeta \otimes U(1)_{HCg}$ symmetry, but instead only $U(1)_{HCg}$.

D. $SU(3)_{MC}$ Sector

The fermions in the second set are involved with the breaking of $SU(3)_1 \otimes SU(3)_2$ to the diagonal color subgroup, color $SU(3)_c$. This set contains nonsinglets under only the group $SU(3)_{MC} \otimes SU(3)_1 \otimes SU(3)_2$ (i.e., they all have zero $U(1)_1$ and $U(2)_2$ hypercharges); with respect to this group, the fermions transform as

$$\xi_R^{a,\lambda} \;:\;\; (3,3,1) \tag{3.20}$$

$$\eta_{p,L}^a \;:\;\; (1,3,1)\,, \quad p = 1,2,3 \tag{3.21}$$

$$\xi_L^{a',\lambda} \;:\;\; (3,1,3) \tag{3.22}$$

and

$$\eta_{p,R}^{a'} \;:\;\; (1,1,3)\,, \quad p = 1,2,3\,, \tag{3.23}$$

where λ, a, and a' are $SU(3)_{MC}$, $SU(3)_1$, and $SU(3)_2$ gauge indices. (This set of fermion fields could be written equivalently in holomorphic form as all right-handed or all left-handed fields by using appropriate complex conjugates.)

The fermions that are nonsinglets under $SU(3)_1$ include the third-generation quarks and the fermions $\xi_R^{a,\lambda}$ and $\eta_{p,L}^a$ in Eqs. (3.20) and (3.21). In order to determine the operative global flavor symmetry involving MC-nonsinglet fermions slightly above 1 TeV, one must take account of the fact that both the $SU(3)_1$ and $SU(3)_{MC}$ interactions become strongly coupled at this scale. In accordance with constraints from custodial symmetry, we shall assume that the $U(1)_1$ gauge coupling is sufficiently weakly coupled so that it, together with the other (non-technicolor) gauge interactions can be neglected, in a leading approximation, in considering the global flavor symmetry. Then the classical global flavor symmetry group at this scale would be

$$U(2)_{(t,b)_L} \otimes U(2)_{(t,b)_R} \otimes U(1)_{\xi_R} \otimes U(3)_{\eta_L} \otimes U(3)_{\xi_L}\,, \tag{3.24}$$

or equivalently,

$$SU(2)_{(t,b)_L} \otimes SU(2)_{(t,b)_R} \otimes U(1)_{(t,b)_V} \otimes U(1)_{(t,b)_A} \otimes U(1)_{\xi_R} \otimes SU(3)_{\eta_L} \otimes U(1)_{\eta_L} \otimes SU(3)_{\xi_L} \otimes U(1)_{\xi_L}\,. \tag{3.25}$$

Here, $SU(2)_{(t,b)_L}$ and $SU(2)_{(t,b)_R}$ operate on the left- and right-handed chiral t and b fields; $U(1)_{(t,b)_V}$ and $U(1)_{(t,b)_A}$ are vector and axial-vector U(1)'s operating on t and b; the $U(1)_{\xi_R}$ rephases the $\xi_R^{a,\lambda}$ fields (for fixed a and λ); the $U(3)_{\eta_L}$ operates on the three $\eta_{p,L}^a$ fields (with a fixed, and $p = 1,2,3$); and the $U(3)_{\xi_L}$ operates on the three $\xi_L^{a',\lambda}$ fields (with λ fixed and $a' = 1,2,3$). $SU(3)_1$ instantons leave $U(1)_{(t,b)_V}$ invariant but break $U(1)_{(t,b)_A}$, $U(1)_{\xi_R}$, and $U(1)_{\eta_L}$. $SU(3)_{MC}$ instantons break the $U(1)_{\xi_R}$ and $U(1)_{\xi_L}$ symmetries. (Thus, in particular, the $U(1)_{\xi_R}$ symmetry is broken by both $SU(3)_1$ and $SU(3)_{MC}$ instantons.) From the broken $U(1)_{(t,b)_A}$ and $U(1)_{\eta_L}$ symmetries one can form a linear combina-

tion, which we denote $U(1)'$, that is preserved by the $SU(3)_1$ instantons. To form a conserved axial-vector current involving the ξ_R field, one needs to cancel the divergences due to both the $SU(3)_1$ and $SU(3)_{MC}$ instantons, which requires a linear combination of the axial-vector currents involving the η_L and ξ_L, respectively. We denote this conserved global symmetry as $U(1)_{scm}$ (where scm = "strongly coupled, mixed"). Thus, with the above-mentioned provisos that other gauge interactions can be considered negligible, the actual quantum global flavor symmetry involving fermions that are nonsinglets under the strongly coupled $SU(3)_1$ and $SU(3)_{MC}$ gauge symmetries is

$$\mathrm{SU(2)}_{(t,b)_L} \otimes \mathrm{SU(2)}_{(t,b)_R} \otimes \mathrm{U(1)}_{(t,b)_V} \otimes \mathrm{SU(3)}_{\eta_L} \otimes \mathrm{U(1)}' \otimes \mathrm{SU(3)}_{\xi_L} \otimes \mathrm{U(1)}_{scm} \ . \tag{3.26}$$

E. $\mathrm{SU(2)}_{UC}$ Sector

The third set of fermions is involved with the breaking of $\mathrm{U(1)}_1 \otimes \mathrm{U(1)}_2$ to the diagonal subgroup, weak hypercharge $\mathrm{U(1)}_Y$. This set contains nonsinglets under only the group $\mathrm{SU(2)}_{UC} \otimes \mathrm{U(1)}_1 \otimes \mathrm{U(1)}_2$, and, with respect to this group, the fields transform as

$$\Omega_R^{\hat{\alpha}} \ : \quad 2_{y,0} \tag{3.27}$$

$$\omega_{p,R} \ : \quad 1_{-y,0} \ , \quad p = 1,2 \tag{3.28}$$

$$\tilde{\Omega}_R^{\hat{\alpha}} \ : \quad 2_{0,-y} \tag{3.29}$$

and

$$\tilde{\omega}_{p,R} \ : \quad 1_{0,y} \ , \quad p = 1,2 \ , \tag{3.30}$$

where $\hat{\alpha}$ is an $\mathrm{SU(2)}_{UC}$ gauge index, the subscripts denote the hypercharges with respect to $\mathrm{U(1)}_1$ and $\mathrm{U(1)}_2$, and $y \neq 0$. This $\mathrm{SU(2)}_{UC}$ sector has a classical global $\mathrm{U(1)}_\Omega \otimes \mathrm{U(1)}_{\hat{\Omega}}$ symmetry, where $\mathrm{U(1)}_{\hat{\Omega}}$ and $\mathrm{U(1)}_{\tilde{\Omega}}$ rephase the $\Omega_R^{\hat{\alpha}}$ and $\tilde{\Omega}_R^{\hat{\alpha}}$ fields, respectively. Both of these U(1)'s are broken by the $\mathrm{SU(2)}_{UC}$ instantons, but the combination corresponding to $N_{UCg} = N_\Omega - N_{\hat{\Omega}}$ is preserved. We denote this as $\mathrm{U(1)}_{UCg}$.

As is evident from these fermion representation assignments, we have chosen to construct the ultraviolet extension to have a modular structure, in which one sector is responsible for the breaking of $\mathrm{SU(3)}_1 \otimes \mathrm{SU(3)}_2$ to $\mathrm{SU(3)}_c$ and another is responsible for the breaking of $\mathrm{U(1)}_1 \otimes \mathrm{U(1)}_2$ to $\mathrm{U(1)}_Y$, rather than trying to accomplish this breaking with a single sector. While this makes the model somewhat complicated, it actually simplifies some aspects of the analysis, such as checking anomaly cancellation and determining condensation channels. One could also investigate models in which one tries to use a single sector to carry out both of these symmetry breakings.

F.   Anomaly Cancellation

Since the full model is a chiral gauge theory, it is necessary to check that it is free of any gauge or global anomalies. Given the modular construction of the theory, we can divide the analysis of anomalies into several parts. The first involves contributions of fermions that are nonsinglets under the $\mathrm{SU(4)}_{ETC}$ group. We first observe that the $\mathrm{SU(4)}_{ETC}$ anomalies of the $\psi_{i,R}$ and $\chi_R^{i,\alpha}$ fields are equivalent to the anomaly of one right-handed fermion in the fundamental representation of $\mathrm{SU(4)}_{ETC}$. This plays the role of a right-handed electroweak-singlet neutrino-type ETC multiplet, so that, in conjunction with the fermion fields in Eqs. (3.5), (3.7), (3.9), (3.11), and (3.13), it renders the part of $\mathrm{SU(4)}_{ETC}$ involving SM-nonsinglet fermions vectorlike, so the $[\mathrm{SU(4)}_{ETC}]^3$ anomaly from these fermions vanishes. The hypercolor sector is constructed so that its contribution to the $[\mathrm{SU(4)}_{ETC}]^3$ anomaly also vanishes, so the entire $[\mathrm{SU(4)}_{ETC}]^3$ anomaly is zero. The anomalies of the form

$$[\mathrm{SU(4)}_{ETC}]^2 \, \mathrm{U(1)}_1, \quad [\mathrm{SU(2)}_L]^2 \, \mathrm{U(1)}_1, \quad \mathrm{and} \quad [\mathrm{U(1)}_1]^3 \tag{3.31}$$

cancel between (quarks plus techniquarks) and (leptons plus technileptons). One also verifies that the following anomalies vanish:

$$[\mathrm{SU(4)}_{ETC}]^2 \, \mathrm{U(1)}_2, \quad [\mathrm{SU(2)}_L]^2 \, \mathrm{U(1)}_2,$$
$$[\mathrm{SU(3)}_1]^3, \quad [\mathrm{SU(3)}_2]^3, \quad [\mathrm{SU(3)}_{MC}]^3$$
$$[\mathrm{SU(3)}_1]^2 \, \mathrm{U(1)}_1, \quad [\mathrm{SU(3)}_1]^2 \, \mathrm{U(1)}_2,$$
$$[\mathrm{SU(3)}_2]^2 \, \mathrm{U(1)}_1, \quad [\mathrm{SU(3)}_2]^2 \, \mathrm{U(1)}_2,$$
$$[\mathrm{SU(3)}_{MC}]^2 \, \mathrm{U(1)}_1, \quad [\mathrm{SU(3)}_{MC}]^2 \, \mathrm{U(1)}_2,$$
$$[\mathrm{SU(2)}_{HC}]^2 \, \mathrm{U(1)}_1, \quad [\mathrm{SU(2)}_{HC}]^2 \, \mathrm{U(1)}_2,$$
$$[\mathrm{SU(2)}_{UC}]^2 \, \mathrm{U(1)}_1, \quad [\mathrm{SU(2)}_{UC}]^2 \, \mathrm{U(1)}_2,$$
$$[\mathrm{U(1)}_2]^3, \quad [\mathrm{U(1)}_1]^2 \, \mathrm{U(1)}_2, \quad [\mathrm{U(1)}_2]^2 \, \mathrm{U(1)}_1 \ . \tag{3.32}$$

Several of these anomalies vanish trivially, owing to the fact that the SM fermions have zero $\mathrm{U(1)}_2$ hypercharge and the modular construction of the theory. Within a semiclassical picture that incorporates gravity, one would also require that the mixed gauge-gravitational anomalies $\mathcal{G}^2 \, \mathrm{U(1)}_1$ and $\mathcal{G}^2 \, \mathrm{U(1)}_2$ vanish, where here $\mathcal{G}$ denotes graviton. This requirement is satisfied.

One also must check that there are no global Witten anomalies (associated with the homotopy group $\pi_4(\mathrm{SU(2)}) = \mathbb{Z}_2$). This requires that the number of chiral fermions transforming as doublets under each of the $\mathrm{SU(2)}$ gauge group be even. For the sector of $\mathrm{SU(2)}_L$-nonsinglet fermions, we have $(N_{gen}-1+N_{TC})(N_c+1) = 16$ chiral doublets. For the $\mathrm{SU(2)}_{HC}$ sector we have $N_{ETC} = 4$ (holomorphic) chiral doublets. Finally, for the $\mathrm{SU(2)}_{UC}$ sector we have $4N_{UC} = 8$ (holomorphic) chiral doublets. Thus, the theory is free of any global π_4 anomaly.

G. Symmetry Breaking of $SU(4)_{ETC}$ to $SU(2)_{TC}$

The model is constructed so that the breaking of the $SU(4)_{ETC}$ symmetry to $SU(3)_{ETC}$ at a scale Λ_1, and then to the residual exact $SU(2)_{TC}$ symmetry at a lower scale Λ_2 is primarily driven by the HC gauge interaction, which is arranged to become strong at a scale $\Lambda_{HC} \simeq \Lambda_1 \sim 10^3$ TeV. The details of how an $SU(4)_{ETC}$ theory can be broken to the residual exact technicolor subgroup $SU(2)_{TC}$ were presented in our Ref. [30], to which we refer the reader. Here we only briefly mention the main points. One chooses the values of the $SU(4)_{ETC}$ and $SU(2)_{HC}$ couplings at a high scale so that at the scale Λ_1, the HC interaction is sufficiently stronger than the ETC interaction that the most attractive channel involves HC-nonsinglet fermions and is of the form (in the notation of Eqs. (3.17)-(3.19))

$$(4,2) \times (6,2) \to (\bar{4},1) \ , \qquad (3.33)$$

with $\Delta C_2 = 5/2$ for $SU(4)_{ETC}$ and $\Delta C_2 = 3/2$ for $SU(2)_{HC}$. The associated condensate is

$$\langle \epsilon_{ijk\ell}\epsilon_{\alpha\beta} \, \chi_R^{j,\alpha \ T}C\zeta_R^{k\ell,\beta}\rangle \ , \qquad (3.34)$$

where $\epsilon_{ijk\ell}$ is the totally antisymmetric tensor density for $SU(4)_{ETC}$. This breaks $SU(4)_{ETC}$ to $SU(3)_{ETC}$ and is invariant under $SU(2)_{HC}$. With no loss of generality, we may define the uncontracted $SU(4)_{ETC}$ index in Eq. (3.34) to be $i = 1$. This condensate also breaks the global $U(1)_{HCg}$ symmetry, giving rise to a Nambu-Goldstone boson (NGB). (Additional physics in a UV completion could render this a PNGB.) In general, (P)NGB's have derivative couplings and hence have interactions that vanish in the limit where the center-of-mass energy $\sqrt{s}$ is much less than the scale of the symmetry breaking, i.e., here, energies much smaller than 10^3 TeV. Moreover, this particular NGB is a SM-singlet, which further suppresses its observable effects. We shall discuss the (pseudo)-Nambu-Goldstone bosons (PNGB's) resulting from the technifermion condensates below.

As the theory evolves to lower energy scales, the $SU(3)_{ETC}$ and $SU(2)_{HC}$ gauge couplings continue to grow, and at the scale Λ_2, the dominant $SU(2)_{HC}$ interaction, in conjunction with the additional strong $SU(3)_{ETC}$ interaction, produces a condensate in the most attractive channel, which is (in a notation analogous to Eq. (3.33))

$$(3,2) \times (3,2) \to (\bar{3},1) \ . \qquad (3.35)$$

This has $\Delta C_2 = 4/3$ for $SU(3)_{ETC}$ and $\Delta C_2 = 3/2$ for $SU(2)_{HC}$. The condensation in this channel breaks $SU(3)_{ETC}$ to $SU(2)_{TC}$ and is invariant under $SU(2)_{HC}$. The associated condensate is

$$\langle \epsilon_{ijk}\epsilon_{\alpha\beta} \, \zeta_R^{1j,\alpha \ T}C\zeta_R^{1k,\beta}\rangle \ , \qquad (3.36)$$

where $i,j,k \in \{2,3,4\}$. With no loss of generality, we may choose $i = 2$ as the breaking direction in $SU(3)_{ETC}$.

Another condensate that is expected to form at a scale slightly below Λ_2 is

$$\langle \epsilon_{\alpha\beta}\chi_R^{1,\alpha \ T}C\zeta_R^{12,\beta}\rangle \ , \qquad (3.37)$$

which does not break any further gauge symmetries beyond those broken at the scales Λ_1 and Λ_2. The choices of $\Lambda_1 \sim 10^3$ TeV and a somewhat smaller value of Λ_2 can yield reasonable values for the masses of the quarks and charged leptons of the first two generations. Details on SM fermion mass generation in this theory are given in Ref. [30]. Since the b quark and τ lepton are $SU(4)_{ETC}$-singlets, they would have to get their masses in a manner different from the quarks and charged leptons of the first two generations. $SU(3)_1$-instanton effects can provide a way to produce m_b (via a 't Hooft determinantal operator) [4]. Further ingredients are required to account for m_τ and to obtain the sort of low-scale seesaw mechanism that was developed in Ref. [20] to explain light neutrino masses (in a full $SU(5)_{ETC}$ theory). Although there are no intrinsic mass terms in the high-scale Lagrangian for the (SM-singlet) fermions (3.17)-(3.19), these fermions all gain dynamical masses of order Λ_1 or Λ_2 as a result of the various condensates that form, and hence are integrated out of the effective low-energy theory below Λ_2.

H. Sequence of Condensations Involving G_{ASM}

The breaking of G_{ASM} is envisioned to occur at a scale roughly of order 1 TeV. In order to analyze this breaking, we first note that at this scale the operative gauge symmetry is the one given in Eq. (2.2). Following usual TC2 practice, the model is arranged so that $\alpha_{c1}(\mu)$ is considerably larger than $\alpha_{c2}(\mu)$ at this scale of about 1 TeV. This inequality in couplings can arise naturally, since the leading coefficient of the $SU(3)_1$ beta function is larger than the corresponding leading coefficient of the $SU(3)_2$ beta function, as a consequence of the fact that the $SU(3)_1$ sector has fewer fermions than the $SU(3)_1$ sector.

With the fermion content as specified above, the $SU(3)_1$ sector has $2 + 3 = 5$ Dirac fermions while the $SU(3)_2$ sector has $8 + 3 = 11$ Dirac fermions, both transforming according to the respective fundamental representations of these two groups. Hence, $b_1 = 23/3$ for $SU(3)_1$ while $b_1 = 11/3$ for $SU(3)_2$. As the energy scale μ decreases through a value denoted Λ_t, the coupling $\alpha_{c1}(\mu)$ grows to be sufficiently large that the $SU(3)_1$ interaction produces a condensate in the $3 \times \bar{3} \to 1$ channel, namely

$$\langle \bar{t}t\rangle = \langle \bar{t}_{a,L}t_R^a\rangle + h.c. \qquad (3.38)$$

This channel has an attractiveness measure $\Delta C_2 = 8/3$ with respect to the $SU(3)_3$ gauge interaction. If one assumes a given value of $\alpha_{c1}(\mu_h)$ at a high scale μ_h, then a rough estimate of the value of the condensation scale Λ_t can be obtained by using Eq. (6.4) in the Appendix. Owing to the formation of the condensate (3.38), the top

quark picks up a dynamical mass, and, indeed, this comprises the dominant part of the mass of the top quark. As discussed above, the $U(1)_1$ interaction is attractive in this channel and repulsive in the $\bar{b}b$ channel. The condensate (3.38) breaks part of the global symmetry group (3.26), namely $SU(2)_{(t,b)_L} \otimes SU(2)_{(t,b)_R}$, to its diagonal subgroup, $SU(2)_{(t,b)_V}$, yielding three (pseudo)-Nambu-Goldstone bosons, $|\pi_t\rangle$. TC2 models require that these be PNGB's rather than strictly massless NGB's because there are also three NGB's $|\pi_{TC}\rangle$ resulting from the formation of the technifermion condensates, with the same SM quantum numbers, and only one set is absorbed to form the longitudinal components of the $W^\pm$ and Z. The $|\pi_t\rangle$ and $|\pi_{TC}\rangle$ mix to form the states

$$|\pi_{EW}\rangle = \cos\theta|\pi_{TC}\rangle + \sin\theta|\pi_t\rangle$$

$$|\pi_T\rangle = -\sin\theta|\pi_{TC}\rangle + \cos\theta|\pi_t\rangle \qquad (3.39)$$

where θ is a mixing angle. This mixing is relatively small, corresponding to the fact noted above that the W and Z masses arise primarily from the technicolor sector. The $|\pi_{EW}\rangle$ are absorbed by the W and Z, while the three orthogonal pseudoscalars $|\pi_T\rangle$ are known as top pions. Using a Gell-Mann-Oakes-Renner-type formula, one infers that the top pions will have masses given by $m_{\pi_T}^2 \simeq -f_t^{-2}m_{t,res}\langle \bar{t}t\rangle$, where $m_{t,res}$ denotes a contribution to m_t that is hard on the scale of Λ_t [5]. This constitutes another necessary ingredient in a satisfactory UV completion of the TC2 model. With Λ_t and f_t as determined, $m_{\pi_T} \sim O(10^2)$ GeV.

A channel of the same $3 \times \bar{3} \to 1$ type with respect to $SU(3)_1$ and thus also a most attractive channel with respect to this group, is one that would break the $SU(3)_{MC}$ gauge symmetry, namely one that would produce the condensates

$$\langle \bar{\eta}_{a,p,L}\xi_R^{a,\lambda}\rangle \;, \quad p = 1,2,3 \;. \qquad (3.40)$$

This condensate also breaks the global $SU(3)_{\eta_L}$ and $U(1)$ symmetries in Eq. (3.26), giving rise to (P)NGB's and also producing dynamical masses of order Λ_t for the $\eta_{p,L}^a$ fields. As before, the (P)NGB's are SM-singlets and are derivatively coupled, so their effects at scales far below 1 TeV are suppressed. These effects merit further study. We assume that if the vacuum alignment is such that the condensate (3.40) does form, it does so at a scale somewhat below $\Lambda_t \simeq \Lambda_{MC}$ and hence does not significantly weaken the effective $SU(3)_{MC}$ interaction at the scale Λ_{MC}. The formation of the condensate (3.40) does have a positive role, since if this did not happen, then the resultant low-energy effective field theory operative below Λ_{MC} would contain three light Dirac color-triplet, electroweak-singlet fermions constructed from the six chiral fermions $\eta_{p,L}^a$ and $\eta_{p,R}^{a'}$, $p = 1,2,3$. Experimentally, such states are excluded, with lower limits of order several hundred GeV, depending on details of the signatures of the production and decays GeV [11, 31].

Written in vectorlike form, the $SU(3)_{MC}$ gauge interaction has three Dirac fermions transforming as the fundamental representation of this group. This number is well below the estimated critical number $N_{f,cr} \simeq 12$ beyond which the theory would evolve into the infrared without spontaneously breaking chiral symmetry (see Appendix for further discussion). It follows that, as the reference scale μ decreases through a scale denoted Λ_{MC}, the $SU(3)_{MC}$ interaction gets sufficiently strong to cause condensation in the channel $3 \times \bar{3} \to 1$, which is the most attractive channel, with condensate

$$\langle \bar{\xi}_{a',\lambda,L}\xi_R^{a,\lambda}\rangle + h.c. \;, \qquad (3.41)$$

breaking $SU(3)_1 \otimes SU(3)_2$ to the diagonal subgroup, $SU(3)_c$. This condensation channel has an attractiveness measure $\Delta C_2 = 8/3$ with respect to the $SU(3)_{MC}$ gauge interaction. The fermions involved in this condensate get dynamical masses of order Λ_{MC} and the gauge bosons (often called colorons) in the coset $[SU(3)_1 \otimes SU(3)_2]/SU(3)_c$ gain masses $(g_{c1}^2 + g_{c2}^2)^{1/2}\Lambda_{MC} \simeq g_{c1}\Lambda_{MC} \simeq \Lambda_{MC}$ (where the running couplings g_{c1} and g_{c2} are evaluated at Λ_{MC}). The model is arranged so that $\Lambda_{MC} \lesssim \Lambda_t$; this is necessary since if Λ_{MC} were larger than Λ_t, then the $SU(3)_1$ would already have broken to regular color $SU(3)_c$, which has a considerably weaker coupling (given by Eq. (3.42) below) at the scale Λ_{MC} and hence would not produce a $\langle \bar{t}t\rangle$ condensate. On the other hand, Λ_{MC} should be high enough so that (i) the colorons have sufficiently large masses to avoid conflict with the lower bounds from experimental searches [9–11] and also (ii) high enough to avoid transitional threshold effects in high-precision Z decay data, which are consistent with equal color $SU(3)_c$ couplings of gluons to the third-generation b quark as well as first- and second-generation quarks. A choice that plausibly satisfies these requirements is $\Lambda_{MC} \sim 1$ TeV. The resultant color $SU(3)_c$ coupling is given by

$$\frac{1}{\alpha_c(\mu)} = \frac{1}{\alpha_{c1}(\mu)} + \frac{1}{\alpha_{c2}(\mu)} \;, \qquad (3.42)$$

so that, with $\alpha_{c1}(\mu) >> \alpha_{c2}(\mu)$, the value of $\alpha_c(\mu)$ at and below Λ_{MC} is set by the weaker coupling, $\alpha_{c2}(\mu)$, and can thus agree with measured values of α_c for $\mu \leq m_Z$. In the effective low-energy field theory operative at scales below Λ_{MC}, we shall use the index a for the resultant $SU(3)_c$ color symmetry. The $SU(3)_{MC}$-induced condensate (3.41) breaks a further part of the (abelian subgroup of the) global symmetry group (3.26).

A comment is in order here concerning the choice of the metacolor gauge group. One might ask why we do not use $SU(2)_{MC}$ rather than $SU(3)_{MC}$. To see why, let us imagine replacing $SU(3)_{MC}$ with $SU(2)_{MC}$, and thus using $p = 1,2$ rather than $p = 1,2,3$ in Eqs. (3.21) and (3.23). Then as the $SU(2)_{MC}$ interaction becomes strong enough to cause condensation, the channel $2 \times 2 \to 1$ leading to the condensate (3.41) would have attractiveness measure $\Delta C_2 = 3/2$. However, in this hypothetical case where the metacolor group is $SU(2)_{MC}$, there would be an even more attractive condensation channel than

the one producing the desired condensate (3.41), namely the channel producing the undesired condensate

$$\langle \epsilon_{abc}\epsilon_{\lambda\rho}\xi_R^{a,\lambda\ T}C\xi_R^{b,\rho}\rangle \ , \qquad (3.43)$$

where a, b, c are SU(3)$_1$ gauge indices and λ, ρ are indices for the hypothetical SU(2)$_{MC}$ group. This condensate is undesired because it would break SU(3)$_1$ to SU(2)$_1$, and this, in turn, would prevent the theory from yielding the usual color SU(3)$_c$ group at lower scales. With the hypothetical SU(2)$_{MC}$ gauge interaction, this undesired condensation channel would have the same degree of attractiveness $\Delta C_2 = 3/2$ with respect to SU(2)$_{MC}$, but it would have an additional attraction due to the SU(3)$_1$ interaction, since it involves the SU(3)$_1$ channel $(3 \times 3)_a \to \bar{3}$, with $\Delta C_2 = 4/3$. Hence, by usual MAC arguments, if the metacolor group were to be taken to be SU(2)$_{MC}$ instead of SU(3)$_{MC}$, the unwanted condensate (3.43) would form first, as the theory evolved toward the infrared, rather than the desired condensate (3.41). The use of SU(3)$_{MC}$ avoids this and guarantees that the MAC is the one that produces the condensate (3.41).

We next proceed to analyze the condensate produced by the SU(2)$_{UC}$ interaction. The SU(2)$_{UC}$ sector is asymptotically free and, when written in vectorial form, contains two Dirac fermions transforming as the fundamental representation. This is well below the estimated critical number, $N_{f,cr} \simeq 8$ (see Appendix) beyond which this theory would evolve into the infrared in a chirally symmetric manner. Hence, as the scale μ decreases from large values through a value $\Lambda_{UC} \simeq \Lambda_{MC}$, the coupling $\alpha_{UC}(\mu)$ grows large enough to trigger condensation in the channel $2 \times 2 \to 1$, with condensate

$$\langle \epsilon_{\hat{\alpha}\hat{\beta}}\Omega_R^{\hat{\beta}\ T}C\tilde{\Omega}_R^{\hat{\alpha}}\rangle + h.c. \qquad (3.44)$$

This breaks the U(1)$_1$ $\otimes$ U(1)$_2$ symmetry to the diagonal subgroup, U(1)$_Y$. The fermions $\Omega_R^{\hat{\alpha}}$ and $\tilde{\Omega}_R^{\hat{\beta}}$ involved in this condensate gain dynamical masses of order Λ_{UC}. The condensate (3.44) preserves the global U(1)$_{UCg}$ symmetry, which is, indeed, the global limit of the U(1)$_Y$ symmetry acting on the UC-nonsinglet fermions. We denote the gauge fields corresponding to U(1)$_1$ and U(1)$_2$ as B_1 and B_2 (suppressing Lorentz indices). The linear combination

$$B' = \frac{-g_1'B_1 + g_2'B_2}{\sqrt{g_1'^2 + g_2'^2}} \qquad (3.45)$$

gains a mass given by

$$m_{B'}^2 \simeq \frac{(g_1'^2 + g_2'^2)y^2\Lambda_{UC}^2}{4} \ . \qquad (3.46)$$

The field (3.45) corresponds to the coset [U(1)$_1$ $\otimes$ U(1)$_2$]/U(1)$_Y$. (The B' is often called by the generic name Z' in the literature.) The orthogonal linear combination,

$$B = \frac{g_2'B_1 + g_1'B_2}{\sqrt{g_1'^2 + g_2'^2}} \qquad (3.47)$$

is the gauge boson corresponding to the usual weak hypercharge U(1)$_Y$ in the SM gauge group G_{SM}, which is massless at this stage, with squared coupling $\alpha'(\mu) = g'(\mu)^2/(4\pi)$ given by

$$\frac{1}{\alpha'(\mu)} = \frac{1}{\alpha_1'(\mu)} + \frac{1}{\alpha_2'(\mu)} \ . \qquad (3.48)$$

At scales μ below $\Lambda_{UC} \simeq \Lambda_{MC}$, the effective gauge group resulting from G_{ASM} that is operative is thus G_{SM}. The residual symmetry resulting from G_{ETCA} includes exact SU(2)$_{TC}$, SU(2)$_{HC}$, and SU(2)$_{UC}$. (The symmetry SU(2)$_{MC}$ may be broken by condensates of the form (3.40).) Without further ingredients, the fermions $\omega_{p,L}$ and $\omega_{p,R}$ with $p = 1, 2$ would remain in this low-energy theory, as a vectorial pair of SU(2)$_L$-singlets with nonzero weak hypercharge. This would be problematic, since, from the standard relation $Q = T_{3L} + (Y/2)$, it follows that they are electrically charged. Such light, charged leptons are excluded for values of $y \sim O(1)$, and $|y|$ cannot be made small compared to unity, because this would reduce the B' mass too much. It is possible that both this problem and the problem of a nearby Landau singularity could be cured by the physics that would involve the embedding of the U(1)$_1$ $\otimes$ U(1)$_2$ symmetry in non-Abelian symmetry group(s) operative at higher energy scales.

I. SU(2)$_{TC}$ Technicolor Sector

The final stage of condensation and resultant electroweak symmetry breaking occurs at the scale Λ_{TC}, where the SU(2)$_{TC}$ interaction becomes sufficiently strongly coupled to produce technifermion condensates, breaking SU(2)$_L$ $\otimes$ U(1)$_Y$ to U(1)$_{em}$. We have discussed this in general above. The resultant W mass is given by Eq. (2.7). A value of Λ_{TC} consistent with $f_{TC} \simeq 120$ GeV is $\Lambda_{TC} \simeq 250$ GeV. With an SU(2)$_{TC}$ gauge group and $2N_{TD} = 2(N_c + 1) = 8$ Dirac technifermions, the technicolor sector can plausibly produce technifermion condensates $\langle \bar{F}F \rangle$ (where F refers to the techni-up quarks, techni-down quarks, technineutrinos, and techni-charged leptons) and exhibit walking behavior associated with an approximate infrared fixed point of the TC beta function [23, 24, 32]. As mentioned above, this walking behavior plays an important role in enhancing the values of SM fermion masses produced by ETC (thereby enabling the theory to use higher ETC scales Λ_j which reduce flavor-changing neutral-current effects) and in making possible a reduction in TC corrections to W and Z propagators, as measured by the S parameter [23, 32–34]. The $|\pi_{EW}\rangle$ of Eq. (3.39) are absorbed to become the longitudinal components of the $W^{\pm}$ and Z. This relies in part on the property that $\Lambda_{TC} << \Lambda_{UC}$, so that the massive Z boson is the same linear combination, in terms of the U(1)$_Y$ gauge field and the neutral SU(2)$_L$ gauge field as in the Standard Model, namely $Z = \cos\theta_W B - \sin\theta_W A_3$. Precision electroweak data

constrain deviations from this form and hence set a lower bound of the TeV order on Λ_{UC}. From these constraints and related constraints on new sources of custodial symmetry violation, it follows that m_Z is given by the SM formula $m_Z = m_W / \cos\theta_W$. In order for the technicolor sector to be in agreement with precision electroweak data, it is necessary that the walking behavior reduce its contributions to the S parameter substantially relative to the value for a QCD-like theory [34]. It is also necessary that the masses of the pseudo-Nambu-Goldstone bosons generated by the chiral symmetry breaking due to the technifermion condensates should be sufficiently large to evade current experimental limits. The walking behavior has been shown to help in this regard [23], but these PNGB's are still a significant concern in this type of ETC model.

IV. ON THE ROLE OF SU(2)$_L$

The group G_{ASM} involves modifications of two of the three factor groups of the Standard Model, leaving the $SU(2)_L$ unmodified. It is natural to ask what the consequences would be of following an analogous procedure with this group, replacing it with the direct product $SU(2)_{L,1} \otimes SU(2)_{L,2}$ in a symmetry group operative at a scale well above 1 TeV. We find that this does not produce the desired $\langle \bar{t}t \rangle$ condensate. To see what would happen, let us first consider a model with the high-scale symmetry group G, as before, but with G_{ASM} replaced by

$$G_{ASM2} = SU(3)_c \otimes SU(2)_{L,1} \otimes SU(2)_{L,2} \otimes U(1)_Y .$$
$$(4.1)$$

The SM-nonsinglet fermions are taken to be as follows, where the numbers in parentheses are the dimensions of the representations of $SU(4)_{ETC}$ and the three non-Abelian factor groups in G_{ASM2}, and the subscript is the weak hypercharge (all of these fields are singlets under $SU(2)_{HC} \otimes SU(3)_{MC} \otimes SU(2)_{UC}$):

$$Q_L^{a,i,k'} : \quad (4,3,1,2)_{1/3} \qquad (4.2)$$

$$\tilde{Q}_L^{a,k} = \begin{pmatrix} t^a \\ b_L^a \end{pmatrix}_L : \quad (1,3,2,1)_{1/3} \qquad (4.3)$$

$$u_R^{a,i} : \quad (4,3,1,1)_{4/3} \qquad (4.4)$$

$$t_R^a : (1,3,1,1)_{4/3} \qquad (4.5)$$

$$d_R^{a,i} : \quad (4,3,1,1)_{-2/3} \qquad (4.6)$$

$$b_R^a : \quad (1,3,1,1)_{-2/3} \qquad (4.7)$$

$$L_L^{k'} : \quad (4,1,1,2)_{-1} \qquad (4.8)$$

$$\tilde{L}_L^k = \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L : \quad (1,1,2,1)_{-1} \qquad (4.9)$$

$$e_R^i ; \quad (4,1,1,1)_{-2} \qquad (4.10)$$

and

$$\tau_R : \quad (1,1,1,1)_{-2} . \qquad (4.11)$$

Here a, i, k, and k' are the $SU(3)_c$, $SU(4)_{ETC}$, $SU(2)_1$, and $SU(2)_2$ gauge indices. We shall, for the moment, leave the SM-singlet sector unspecified; we comment on this later. All of the non-Abelian gauge interactions here are asymptotically free.

We assume that the values of the corresponding gauge couplings are such that, as the energy scale μ decreases, the first interaction to become sufficiently strongly coupled to form a condensate is $SU(2)_{L,1}$. Let us denote the scale at which this occurs as Λ_L. We note that the $SU(2)_{L,1}$ sector, as specified so far, has $N_c + 1 = 4$ chiral SM-nonsinglet fermions, or equivalently, two Dirac fermions, so that it is well within the phase where chiral symmetry breaking takes place in the infrared. However, the resultant condensates of third-generation fermions are not of the desired type. The most attractive channel for the strongly coupled $SU(2)_{L,1}$ interaction is $2 \times 2 \to 1$, and it produces several condensates in this channel. The first of these is of the form $\langle \epsilon_{k\ell} \tilde{Q}_L^{a,k \, T} C \tilde{Q}_L^{b,\ell} \rangle$, where $\epsilon_{k\ell}$ is the antisymmetric tensor density for $SU(2)_{L,1}$. This is automatically antisymmetric in $SU(3)_c$ indices and hence is proportional to

$$\langle \epsilon_{abc} \epsilon_{k\ell} \tilde{Q}_L^{a,k \, T} C \tilde{Q}_L^{b,\ell} \rangle = 2 \langle \epsilon_{abc} t_L^{a \, T} C b_L^b \rangle , \qquad (4.12)$$

where ϵ_{abc} is the antisymmetric tensor density of $SU(3)_c$. This transforms as a $(3 \times 3)_{antisym} = \bar{3}$ under $SU(3)_c$ and hence breaks $SU(3)_c$ to a subgroup $SU(2)_c$. It also violates hypercharge and electric charge. The strong $SU(2)_1$ interaction would also produce the condensate

$$\langle \epsilon_{k\ell} \tilde{Q}_L^{a,k \, T} C \tilde{L}_L^\ell \rangle = \langle t_L^{a \, T} C \tau_L - b_L^{a \, T} C \nu_{\tau L} \rangle , \qquad (4.13)$$

which also breaks $SU(3)_3$ to an $SU(2)_c$ subgroup and violates hypercharge and electric charge. In addition to breaking these gauge symmetries, the condensate (4.12) breaks baryon number by $\Delta B = 2/3$, while the condensate (4.13) breaks B by $\Delta B = 1/3$ and lepton number L by $\Delta L = 1$. For all of these reasons, one avoids trying to construct a model with $SU(2)_L$ replaced by $SU(2)_{L,1} \otimes SU(2)_{L,2}$.

V. CONCLUSIONS

In conclusion, we have carried out an exploratory construction and analysis of an ultraviolet extension of a

TC2 theory. Let us assess the merits and shortcomings of this model. Among the partial successes is the fact that our model includes self-consistent dynamical mechanisms that can account for (i) the breaking of the ETC gauge symmetry down to the residual exact technicolor symmetry, (ii) the breaking of $SU(3)_1 \otimes SU(3)_2$ to $SU(3)_c$, and (iii) the breaking of $U(1)_1 \otimes U(1)_2$ to $U(1)_Y$. The model incorporates the characteristic feature of topcolor and TC2 theories, that the large mass of the top quark arises from the formation of a $\langle \bar{t}t \rangle$ condensate. Given that the resultant technicolor sector plausibly has a large but slowly running gauge coupling governed by an approximate infrared fixed point, the TC/ETC interactions can produce a reasonable spectrum of masses for the first two generations of quarks and charged leptons.

However, the model needs additional ingredients to be more realistic. Although we are able to embed two of the three SM fermion generations in an ETC framework, the model cannot contain a full embedding of all three generations in the ETC symmetry in the usual manner because the generational part of the ETC symmetry is incompatible with the property that the first two generations of SM fermions transform differently under G_{ASM} from the third generation. Consequently, this model only has mixing between the first two generations. A fully satisfactory explanation of the τ lepton mass and the masses and mixings of neutrinos requires further ingredients, as does an explanation of the contribution to m_t that is hard on the scale Λ_t. Moreover, the model contains the non-asymptotically free $U(1)_1 \otimes U(1)_2$ sector of topcolor and TC2 models. As usual in TC2 models, the strength of the $U(1)_1$ interaction is bounded above both by its violation of custodial symmetry and by the requirement that there not be any nearby Landau pole. One would hope in further work to embed this abelian product group in some asymptotically free, non-Abelian symmetry group(s). The condensates that are formed by the HC and MC interactions entail the appearance of (pseudo)-Nambu-Goldstone bosons, whose phenomenological consequences need further study. Some of the new fermions with nonzero SM quantum numbers, in particular, the color-singlet, $SU(2)_L$-singlet, charged ω and $\tilde{\omega}$ fermions, do not pick up large dynamical masses and hence would remain light, indicating the need for further ingredients to make the model more realistic. A general comment is that the model has a profusion of new gauge interactions. In an ultimate theory one would, of course, need to explain the relative values of the many gauge couplings. Finally, there are the usual concerns with TC/ETC models such as ensuring that pseudo-Nambu-Goldstone bosons that result from the technifermion condensates are sufficiently heavy, and ensuring that technicolor contributions to the S parameter are sufficiently small to agree with experimental constraints.

Nevertheless, given the interest in models of electroweak symmetry breaking involving both technifermion and top-quark condensates, we believe that it is valuable to carry out this type of exploratory investigation of pos-sible ultraviolet extensions. It is hoped that the present study will be of use in elucidating some of the challenges that one faces in building ultraviolet completions of TC2 theories.

VI. APPENDIX

In this Appendix we list some results used in the text. The beta function of a given factor group G_j with running gauge coupling $g_j(\mu)$ is $\beta_j = dg_j/dt$, where $t = \ln \mu$ and μ is the reference scale. In terms of α_j,

$$\frac{d\alpha_j}{dt} = -\frac{\alpha_j^2}{2\pi}\left[(b_1)_j + \frac{(b_2)_j \alpha_j}{4\pi} + O(\alpha_j^2)\right] , \qquad (6.1)$$

where we recall that the first two coefficients, $(b_1)_j$ and $(b_2)_j$, are scheme-independent. The beta function with perturbatively calculated coefficients is appropriate to describe the running of a gauge coupling that is not too large, in the energy range where the corresponding gauge fields are dynamical (i.e., above corresponding scales at which G_j is broken). For the condensation channel (3.15), a solution of the Dyson-Schwinger equation in the approximation of one-gauge boson exchange (often called the ladder approximation) yields the condition for the critical coupling $\alpha_{j,cr}$ (e.g, [23])

$$\frac{3\alpha_{j,cr}\Delta C_2}{2\pi} = 1 , \qquad (6.2)$$

where ΔC_2 was given in Eq. (3.16). Clearly, this is only a rough estimate, in view of the strong-coupling nature of the physics.

To investigate the infrared behavior of an asymptotically free G_j gauge interaction (i.e., one for which $(b_1)_j > 0$), one integrates the beta function. For sufficiently few fermions that are nonsinglets under G_j, $(b_2)_j > 0$ and the coupling will eventually exceed the critical coupling for condensation in some most attractive channel. As the number of fermions that are nonsinglets under G_j increases, the sign of $(b_2)_j$ eventually reverses, and in this case, the (perturbative) two-loop beta function has a zero away from the origin, at

$$\alpha_{j,IR} = -\frac{4\pi(b_1)_j}{(b_2)_j} . \qquad (6.3)$$

Within the context of these approximations, if this $\alpha_{j,IR} > \alpha_{j,cr}$, then as α_j exceeds $\alpha_{j,cr}$, there is condensation in the most attractive channel. In this case, the fermions involved in this condensate gain dynamical masses and the evolution of the theory further into the infrared is governed by a different beta function, so that $\alpha_{j,IR}$ is only an approximate fixed point. If, on

the other hand, $\alpha_{j,IR} < \alpha_{j,cr}$ for any condensation channel, then the G_j sector evolves into the infrared without any fermion condensation or associated spontaneous chiral symmetry breaking and $\alpha_{j,IR}$ is an exact infrared fixed point. In each of these cases, the behavior that occurs as a particular G_j gauge sector evolves toward the infrared can be changed if other interactions break the G_j symmetry, as happens, for example, with $SU(3)_2$.

Moreover, for an $SU(2)$ gauge theory with chiral fermions transforming according to the fundamental representation, the absence of a global Witten anomaly means that there must be an even number of these fermions, so that one can always rewrite the theory in a vectorial form with N_f Dirac fermions. Then the above analysis yields the estimate $N_{f,cr} \simeq 8$ for the critical

number of such fermions below which the theory has SχSB in the infrared [24]. For a vectorial (asymptotically free) $SU(3)$ theory with N_f Dirac fermions transforming according to the fundamental representation, the same type of analysis yields the estimate $N_{f,cr} \simeq 12$. Recent lattice results for $SU(3)$ are broadly consistent, to within the theoretical uncertainties, with this estimate [32].

Integrating Eq. (6.1) and imposing the condition (6.2) yields, to leading order, the following rough estimate for the energy scale where this condensation occurs:

$$\mu_{c,j} \simeq \mu_h \exp\left[-\frac{2\pi}{(b_1)_j} \left(\alpha_j(\mu_h)^{-1} - \frac{3\Delta C_2}{2\pi} \right) \right] , \quad (6.4)$$

where μ_h is a high-energy reference scale.

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